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On the Feynman rules for chiral interactions

Peter T Davies

Department of Physics, Queen Mary College, University of London, Mile End Road, London E1 4NS, UK

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Abstract. We consider the steps necessary to set up a perturbation theory for the chiral invariant self-interaction of massless bosons. In each step we find something different to what we might naively expect from past experience of other interactions, but we can decide what the canonical procedures give and then, wherever possible, replace the canonical procedures by the more naive Feynman rules and see what compensation has to be introduced into the interaction.

We start with the ordered hamiltonian given by Charap, and find that its use is equivalent to the use of the expected $(-\mathscr{L}_{\text{int}})$ term, but modified by two singular terms. The first, being proportional to $\delta^4(0)$, is well known. The second, proportional to $(\delta^3(0))^2$, comes from the correct use of Wick's theorem with our specified ordering. We also find that we cannot use normal ordering to rid ourselves of tadpole contributions, though this is not because the required normal ordering violates the chiral invariance. It is a characteristic of any such self-interaction theories and therefore has important consequences for many superpropagator calculations.

1. Introduction

Following the developments in nonpolynomial field theory has come the suggestion that chiral invariant lagrangians could perhaps become the basis of a perturbative, dynamical field theory. This is a departure from their more common use with the tree-graph approximation as 'effective' lagrangians, that is, solely a calculation device for yielding current algebra results. The use of a nonlinear realization of the chiral symmetry on pseudoscalar meson fields makes the interaction nonpolynomial in those fields and it may be that superpropagator techniques can be used to specify otherwise arbitrary renormalization constants (Lehmann and Trute 1972, Ecker and Honerkamp 1972, Abdus Salam 1971). Accepting that there is some motivation for looking at a perturbation series we must ask: what are the Feynman rules to be used in calculating the amplitudes? This is the question to which we address ourselves for the self-interaction of massless pseudoscalar mesons. Many of the problems will be general features of any two-derivative polynomial, or nonpolynomial, interaction and we expect to see similar problems, with similar resolution, arise in the proper treatment of quantum gravity.

The usual steps in setting up a perturbation theory are:

(i) Construct a classical hamiltonian from the classical lagrangian and impose canonical commutation relations to give a quantum mechanical hamiltonian in the Heisenberg picture.

(ii) Change from the Heisenberg to the interaction picture (or from interpolating fields to in/out fields if we follow the LSZ formalism) through a unitary transformation, and hence to the Feynman–Dyson solution for the S matrix.

(iii) Use Wick's theorem to expand time-ordered products.
(iv) Use Matthews' T^* ordering and replace $\mathscr{H}_{\text{int}}$ by $-\mathscr{L}_{\text{int}}$ to exhibit manifest Lorentz covariance when we have derivative couplings.
(v) Rid ourselves of tadpole (closed loops starting and finishing at the same space–time point) contributions by normal ordering.

Now, our chiral interaction has two derivative couplings and is a nonpolynomial function of the fields and, when we check the application of the above five steps, we find that something is different in every case from what we might naively expect on the basis of experience with better known interactions; for example, of the type $e\bar{\psi}\psi A$ or $g\bar{\psi}\psi\partial_\mu\phi$. To be more specific, the quantum hamiltonian has an ordering ambiguity due to the presence of noncommuting factors, the transformation of (ii) may take us from a nonlinear function of the fields to a linear function, proper use of Wick's theorem involves Wightman functions as well as the expected Feynman propagators, and the T^* theorem does not apply. In some cases the departure from a naive treatment has been previously noted: the correct use of T^* ordering has been made by many authors (Charap 1971, Gerstein *et al* 1971) and steps (ii) and (iii) have been considered by Okabayashi *et al* (1972) and Suzuki and Hattori (1972) respectively though the ordering problem has either been overlooked or not treated correctly.

This ordering has now been specified by Charap (1973) who demands that it be consistent with chiral invariance of the hamiltonian. We will trace the effect that the specified ordering has on each of the steps above. Wherever possible, having seen what the canonical procedures give us, we will then adopt the more common, naive procedures and see what change in the interaction we need to compensate for this. That is, we are aiming for the interaction to use when we ignore the ordering, make naive use of Wick's theorem and use T^* ordering, such that the results are those which derive from the correct canonical procedures. We find that the naive interaction $-\mathscr{L}_{\text{int}}$ is modified by two highly singular terms. The first, which is proportional to $\delta^4(0)$, is well known and is a result of the T^* ordering prescription. The second, proportional to $(\delta^3(0))^2$ is a result of the effect of the specified ordering upon the use of Wick's theorem.

Perhaps the most surprising result concerns the normal ordering. Contrary to an often repeated assertion, we find that it is not the chiral invariance and its consequent specified ordering which stops us normal ordering. We cannot normal order because we have only one coupling constant, which is a common property of all self-interactions of this type. This inability to rid ourselves of the singular tadpole diagrams has important consequences for the many superpropagator calculations where normal ordering has been assumed.

2. The ordered hamiltonian

The work of Charap (1973) and Parish (1973) specifies for us an energy–momentum tensor which has the Sugawara current–current form. The ordering of noncommuting factors is specified uniquely (up to a unitary transformation of all the operators) by asking that it be consistent with chiral invariance. We shall be content here with quoting the ordered hamiltonian density which follows.

$$\hat{\mathscr{H}}(x) = \tfrac{1}{8}\lambda^2\{\hat{\pi}_i, \hat{F}^{ai}\}\{\hat{\pi}_j, \hat{F}^{aj}\} + \tfrac{1}{2}\nabla\hat{\phi}^i\hat{g}_{ij}\nabla\hat{\phi}^j \tag{2.1}$$

where the carets indicate that the fields $\hat{\phi}_i(x)$, and their conjugate momenta $\hat{\pi}_i(x)$, are in the Heisenberg picture.

$$\hat{\pi}_i = \tfrac{1}{2}\{\hat{g}_{ij}, \dot{\hat{\phi}}^j\} \tag{2.2}$$

where $i,j = 1,\ldots,8$ if the chiral group is $SU(3)\times SU(3)$, and $i,j = 1,\ldots,3$ for $SU(2)\times SU(2)$.

$$\lambda^2 \hat{F}^{ai}\hat{F}^{aj} = (\hat{g}^{-1})_{ij} \tag{2.3}$$

where $a = 1,\ldots,16$ for $SU(3)\times SU(3)$ and $a = 1,\ldots,6$ for $SU(2)\times SU(2)$. The canonical equal-time commutator between $\hat{\pi}$ and $\hat{\phi}$ is:

$$[\hat{\pi}_i(\boldsymbol{x},t), \hat{\phi}_j(\boldsymbol{y},t)] = -\mathrm{i}\delta_{ij}\delta^3(\boldsymbol{x}-\boldsymbol{y}). \tag{2.4}$$

The dimensional coupling constant $\lambda = f_\pi^{-1}$, the inverse of the unrenormalized pion decay constant; and $\hat{g}_{ij}$ is the chiral invariant metric tensor which is in general a non-polynomial function of the fields. We note that, if we ignore the ordering, then $\hat{\mathscr{H}}(x)$ would be the classical hamiltonian that follows from the following classical lagrangian:

$$\mathscr{L}(x) = \tfrac{1}{2}\partial_\mu\hat{\phi}^i\hat{g}_{ij}\partial^\mu\hat{\phi}^j. \tag{2.5}$$

The functions F^{ai} give the group transformations of the fields

$$[\hat{Q}^a, \hat{\phi}^i(x)] = \mathrm{i}F^{ai}(\hat{\phi}(x)) \tag{2.6}$$

where the $\{\hat{Q}^a\}$ are the axial and vector generators. We make no distinction between upper and lower group indices.

Having taken this result we prepare for a perturbation theory by casting the interaction into the interaction picture, we will denote this by operators without carets. As usual we assume the existence of a unitary transformation $U[\sigma,\sigma_0]$, with an initial condition on some space-like surface σ_0, $U[\sigma_0,\sigma_0] = 1$, such that

$$\phi_i(x) = U[\sigma,\sigma_0]\hat{\phi}_i(x)U^{-1}[\sigma,\sigma_0] \qquad x\in\sigma. \tag{2.7}$$

The transformation is specified by asking that it satisfy the Tomonaga–Schwinger equation

$$\mathrm{i}\frac{\delta}{\delta\sigma(x)}U[\sigma,\sigma_0] = \mathscr{H}_{\mathrm{int}}(x)U[\sigma,\sigma_0] \tag{2.8}$$

where

$$\mathscr{H}_{\mathrm{int}} = U\hat{\mathscr{H}}_{\mathrm{int}}U^{-1}. \tag{2.9}$$

The perturbation theory rests on being able to solve for U with a Volterra integral equation, and thus we must check that the integrability condition is satisfied:

$$[\mathscr{H}_{\mathrm{int}}(x), \mathscr{H}_{\mathrm{int}}(y)] = 0 \qquad x,y\in\sigma. \tag{2.10}$$

This is trivially satisfied for space-like σ if there are no derivatives in the interaction, and it is a fairly straightforward matter to check that the presence of powers of the momentum in a specified ordering does not change the result. Note that to do this we need make no assumptions about the form of $\mathscr{H}_{\mathrm{int}}$. Use of (2.9) puts (2.10) into the Heisenberg picture and $\hat{\mathscr{H}}_{\mathrm{int}}$ we know. Having verified (2.10) for space-like σ we can specialize to the hypersurface normal to the time-like vector $\eta_\mu = (1,\mathbf{0})$. Then taking the gradient of the non-covariant form of (2.7) gives

$$\partial_\mu\phi_i(x) = U(t,t_0)\partial_\mu\hat{\phi}_i(x)U^{-1}(t,t_0) - \mathrm{i}\eta_\mu[H_{\mathrm{int}}(t), \phi_i(x)] \tag{2.11}$$

which yields the expected result

$$U\nabla\hat{\phi}_i(x)U^{-1} = \nabla\phi_i(x). \tag{2.12}$$

The transformation of $\dot{\hat{\phi}}_i$ tells us how $\hat{\pi}_i$ transforms and once again we emphasize that we need no assumption about $\mathscr{H}_{\text{int}}$, as we can rewrite the commutator in (2.11) in terms of the known Heisenberg picture operators. With the use of the definition (2.2) we have

$$U\hat{\pi}_i U^{-1} = \dot{\phi}_i. \tag{2.13}$$

Note that this transformation has taken $\hat{\pi}_i$ (nonlinear in the fields) into the linear $\dot{\phi}_i$, but this is just what we require of the interaction picture wherein the operators must satisfy free-field commutation relations.

Thus in the interaction picture we have

$$\mathscr{H}_{\text{int}}(x) = \tfrac{1}{8}\lambda^2\{\dot{\phi}, F\}\{\dot{\phi}, F\} - \tfrac{1}{2}\dot{\phi}\dot{\phi} + \tfrac{1}{2}\nabla\phi\tilde{g}\nabla\phi \tag{2.14}$$

$$\tilde{g} \equiv g - 1$$

where we suppress the group indices if no confusion is likely to occur. Correct use of the transformations (2.7), (2.12) and (2.13) has been previously made by Gerstein *et al* (1971) for the unordered interaction.

3. Use of Wick's theorem

Our Feynman rules come from the expansion in $\mathscr{H}_{\text{int}}$ of such objects as:

$$\left\langle 0 \left| T\Omega(x_1, \ldots, x_n) \exp\left(-\mathrm{i} \int \mathrm{d}^4 y \mathscr{H}_{\text{int}}(y) \right) \right| 0 \right\rangle_{\mathrm{c}} \tag{3.1}$$

for some function Ω of interaction picture fields (or in/out fields if we follow LSZ procedures). The subscript c is telling us to normalize with the vacuum-expectation-value of the S matrix which has the effect of leaving contributions from connected diagrams only. The expansion is facilitated by the use of Wick's theorem which states:

$$T(UV\ldots XY) = :UV\ldots XY: + :U'V'\ldots XY: + \text{perms} + :\text{etc} \tag{3.2}$$

where one commonly assumes that

$$U'V' = \langle 0|T(UV)|0\rangle \tag{3.3}$$

but this is only true if the operators U and V come from different hamiltonians, that is, they have different time arguments. If they belong to the same hamiltonian

$$U'V' = \langle 0|UV|0\rangle \tag{3.4}$$

where the order is as it was in the hamiltonian. With no derivative couplings this is a trivial distinction because the fields commute at the same space–time point and we can consistently use the Feynman propagator (3.3)

$$\langle 0|\phi(x)\phi(x)|0\rangle \equiv \Delta^+(0) = \Delta^{\mathrm{F}}(0). \tag{3.5}$$

But with derivative couplings this is not the case. For instance, if we have the one-derivative tadpole contribution $\dot{\phi}(x)'\phi(x)'$, then the correct use of Wick's theorem with the Wightman function (3.4) gives a singular contribution whereas the naive use with (3.3) gives nothing. That is,

$$\dot{\Delta}^{+}(0) = -\tfrac{1}{2}\mathrm{i}\delta^{3}(0) \tag{3.6a}$$

whereas

$$\dot{\Delta}^{\mathrm{F}}(0) = 0. \tag{3.6b}$$

Thus the correct use of Wick's theorem would appear to give different results for at least one-derivative tadpole diagrams, but the specified ordering modifies this conclusion. We notice that, if our hamiltonian has its noncommuting factors of $\dot{\phi}$ and ϕ symmetrized, then instead of getting a $\dot{\Delta}^{+}(0)$ we would have $\dot{\Delta}^{1}(0)$, which is zero:

$$\dot{\Delta}^{1}(0) \equiv \langle 0|\{\dot{\phi}(x), \phi(0)\}|0\rangle|_{x=0} = 0. \tag{3.7}$$

Thus the result is the same as ignoring ordering and using (3.3). Looking at the ordered form of $\mathscr{H}_{\mathrm{int}}$ given by (2.14) we see that the ordering does not affect two-derivative tadpoles, nonderivative tadpoles or proper exchanges between two vertices, but the only one-derivative tadpole diagrams which escape being zero through (3.7) are the 'rabbit ears' diagrams shown in figure 1, which come from the contractions:

$$\{\dot{\phi}', F''\}\{\dot{\phi}'', F'\}. \tag{3.8}$$

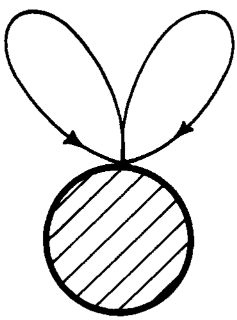

Figure 1. A 'rabbit ears' diagram. The arrows indicate derivative coupling.

These diagrams contribute a $-(\dot{\Delta}^{+}(0))^2 = \frac{1}{4}(\delta^3(0))^2$ and leave behind, for further contractions, $4V(\phi)$ where,

$$V(\phi) \equiv \frac{\lambda^2}{8}\frac{\partial}{\partial\phi_j}F^{ai}\frac{\partial}{\partial\phi_i}F^{aj}. \tag{3.9}$$

Thus, if we: (i) ignore the ordering; (ii) use naive Wick's theorem with consistent use of (3.3); we have to add in the 'rabbit ears' contribution. The compensated interaction $\mathscr{H}^{(1)}_{\mathrm{int}}$ can therefore be written, with use of (2.3), as

$$\mathscr{H}^{(1)}_{\mathrm{int}} = \tfrac{1}{2}\dot{\phi}G\dot{\phi} + \tfrac{1}{2}\nabla\phi\tilde{g}\nabla\phi + (\delta^3(0))^2 V(\phi) \tag{3.10}$$

$$G = g^{-1} - 1.$$

A similar result has been achieved by Suzuki and Hattori (1972), though they did not work with the chiral invariant ordering (see the discussion in Charap 1973). They note that, with their definition of $\theta(0) = \frac{1}{2}$, $\theta(t)$ being the step function,

$$\langle 0|\{U(t), V(t')\}|0\rangle - \langle 0|T\{U(t), V(t')\}|0\rangle|_{t=t'} = 0 \tag{3.11}$$

and thus a term totally symmetrized in its noncommuting factors gives the same results as the naive use of Wick's theorem gives. Indeed, we can use the canonical commutation relations to write

$$\tfrac{1}{8}\lambda^2\{\dot{\phi}, F\}\{\dot{\phi}, F\} = \tfrac{1}{8}\{\dot{\phi}, \{\dot{\phi}, g^{-1}\}\} + (\delta^3(0))^2 V(\phi). \tag{3.12}$$

We note that, when the noncommuting factors are the field and its time derivative, (3.11) is true by virtue of both terms being separately zero whatever the value of $\theta(0)$.

The existence of the $(\delta^3(0))^2$ terms in (3.10) has also been supported by Dowker and Mayes (1971).

4. Manifest covariance

In pursuit of manifest Lorentz covariance we rewrite (3.10) as

$$\mathscr{H}^{(1)}_{\text{int}} = -\mathscr{L}_{\text{int}} + \tfrac{1}{2}\dot{\phi}(\tilde{g}^2 g^{-1})\dot{\phi} + (\delta^3(0))^2 V(\phi) \tag{4.1}$$

where

$$\mathscr{L}_{\text{int}} = \tfrac{1}{2}\partial_\mu\phi\tilde{g}\partial^\mu\phi. \tag{4.2}$$

The time-ordering prescription in the perturbation theory also leads to noncovariant terms:

$$\langle 0|T\partial_\mu\phi(x)\partial_\nu\phi(0)|0\rangle = \mathrm{i}\partial_\mu\partial_\nu\Delta^{\mathrm{F}}(x) - \mathrm{i}\eta_\mu\eta_\nu\delta^4(x). \tag{4.3}$$

With certain types of derivative interaction, for example $\bar{\psi}\gamma^\mu\psi\partial_\mu\phi$, Matthews' theorem (1949) tells us that the noncovariant parts of the hamiltonian and propagator exactly cancel each other to all orders in the S matrix, but here it is well known that this is not the case (Charap 1971, Gerstein *et al* 1971). If we use the T^* ordering prescription

$$\langle 0|T^*\partial_\mu\phi(x)\partial_\nu\phi(0)|0\rangle = \mathrm{i}\partial_\mu\partial_\nu\Delta^{\mathrm{F}}(x) \tag{4.4}$$

then we must replace the second term in (4.1) with a singular, covariant function, to give

$$\mathscr{H}^{(2)}_{\text{int}} = -\mathscr{L}_{\text{int}} + \tfrac{1}{2}\mathrm{i}\delta^4(0)\ln(\det g) + (\delta^3(0))^2 V(\phi). \tag{4.5}$$

This, then, is the compensated interaction to use if we: (i) ignore ordering; (ii) make naive use of Wick's theorem; (iii) use T^* ordering.

There is some evidence from low-order calculations (Charap 1970, Suzuki and Hattori 1972) that there will be no $\delta^4(0)$ or $(\delta^3(0))^2$ contributions to the final S matrix because the two extra terms in (4.5) cancel contributions from the $-\mathscr{L}_{\text{int}}$ part. If this conjecture is true, then we need only use the most naive Feynman rules with $\mathscr{H}_{\text{int}} = -\mathscr{L}_{\text{int}}$ and drop all $\delta^4(0)$ or $(\delta^3(0))^2$ terms whenever they occur. Apart from the problems of normal ordering, we then see some justification for the superpropagator calculations that have been carried out in the past.

5. Tadpole diagrams

We have seen that, if we use the form of the interaction in (4.5), we have no tadpoles appearing which have one derivative in them. It is also true that there are no two-derivative tadpoles, which may not be at first evident because the Feynman propagator satisfies the inhomogeneous Klein–Gordon equation

$$\Box \Delta^{F}(x) = \delta^4(x) \tag{5.1}$$

and this would seem to introduce $\delta^4(0)$ terms. In the canonical theory we know there are no such contributions because the Wightman function satisfies the homogeneous equation

$$\Box \Delta^{+}(x) = 0. \tag{5.2}$$

It is also true that the two-derivative tadpoles do not appear with the use of $\mathscr{H}^{(1)}_{\text{int}}$ because the trace in μ and ν of (4.3) is again zero. Therefore it is the T^* ordering prescription which seems to introduce them, thus it must be the second term in $\mathscr{H}^{(2)}_{\text{int}}$ which cancels them. This is precisely what does happen because

$$\ln \det g = \operatorname{Tr} \ln g = \operatorname{Tr}(\tilde{g} - \tfrac{1}{2}\tilde{g}^2 + \ldots) \tag{5.3}$$

and the $\operatorname{Tr} \tilde{g}$ provides the cancellations that we need.

We are thus left with only the nonderivative tadpoles and the question of normal ordering now arises. We see that we need only normal order the functions of ϕ_i which occur in (4.5) which has *no consequence for the chiral invariance.* It is the equivalent of normal ordering the functions of ϕ_i in (2.14) (when rewritten with the use of (3.12)) which does not spoil the chiral ordering. Initially we might have expected to normal order the whole interaction which would have violated the invariance.

However, despite the easing of this restriction, we find that we still cannot rid ourselves of tadpole contributions. We will illustrate with the self-interaction of a massless, singlet boson field ϕ:

$$\mathscr{H}_{\text{int}} = F(\kappa\phi) \tag{5.4}$$

where F may be a polynomial or nonpolynomial function and κ is a dimensional coupling constant such that $\kappa\phi$ is dimensionless. To rewrite (5.4) such that it is in normal order, we use the formal expression

$$\phi^n = :\exp\left(\frac{1}{2}\Delta\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}\right)\phi^n: \tag{5.5}$$

$$\Delta \equiv \Delta^{+}(0) = \Delta^{F}(0)$$

where we apply the exponential operator on the right-hand side and interpret the resultant terms as being in normal order. In general, then, using the operator on F gives

$$F(\kappa\phi) = :G(\kappa^2\Delta, \kappa\phi): \neq :F(\kappa\phi): + C(\kappa^2\Delta) \tag{5.6}$$

the only exception being the familar case when F is bilinear in the fields. If (5.6) were an equality we could throw away the additive infinite C number because of the connected-diagrams-only prescription in (3.1). As it is, the most that we can ask is that normal ordering does not change the form of our interaction

$$F(\kappa\phi) = C(\kappa^2\Delta):F(\kappa\phi):. \tag{5.7}$$

This can be easily solved to give $F(\kappa\phi) = \exp(\kappa\phi)$ whereby

$$\exp(\kappa\phi) = \exp(\tfrac{1}{2}\kappa^2\Delta):\exp(\kappa\phi):. \tag{5.8}$$

We note that, in the case of the interaction of a multiplet of hermitian fields, $\boldsymbol{\phi} = \{\phi_i\}$ we can make use of

$$\langle 0|\phi_i\phi_j|0\rangle = \delta_{ij}\Delta \tag{5.9}$$

to normal order a general function of the multiplet fields with the operator

$$\exp\left(\frac{1}{2}\Delta\frac{\partial^2}{\partial\boldsymbol{\phi}\,.\,\partial\boldsymbol{\phi}}\right).$$

As an example, if we had an isoscalar function of the pion triplet $\boldsymbol{\pi}$, we could ask for the function that satisfied the equivalent of (5.7) and would find the solution (regular at zero field strength)

$$\frac{\sinh(\kappa\pi)}{\pi} = \exp(\tfrac{1}{2}\kappa^2\Delta):\frac{\sinh(\kappa\pi)}{\pi}: \tag{5.10}$$

where

$$\pi = \sqrt{\boldsymbol{\pi}\,.\,\boldsymbol{\pi}}.$$

Thus, if we have interactions of the form $e\bar{\psi}\psi\exp(\kappa\phi)$ or $g\bar{\psi}\psi\sinh(\kappa\pi)/\pi$ which might serve as prototypes for gravity-modified quantum electrodynamics or chiral invariant pion–nucleon interaction we can absorb the multiplicative infinite C number into a redefinition of the unrenormalized major coupling constant e or g. In the case of a self-interaction with *only one coupling constant*, this is not possible, we are stuck with our infinity, though we have achieved the summing of all tadpole contributions into it. A question for future study is whether or not a proper renormalization program could absorb the infinity into a wavefunction renormalization.

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